

Der Gauß - Algorithmus

Der Algorithmus von Gauss ist das universelle Verfahren zur Lösung beliebiger linearer Gleichungssysteme.

Einführungsbeispiel:

$7x_1 + 3x_2 - 5x_3 = -12$ $-x_1 - 2x_2 + 4x_3 = 5$ $-4x_1 + x_2 - 3x_3 = 1$	Drei Gleichungen mit drei Variablen	Es wird zeilenweise gearbeitet. Zeilen darf man: - vertauschen - mit einer Zahl multiplizieren - durch eine Zahl dividieren - addieren - subtrahieren * bedeutet irgendeine Zahl Spalten dürfen ebenfalls vertauscht werden, wenn die Variable x_i mitgenommen wird																																
Rechenschema:	Die Umformung soll ergeben:																																	
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0; 1 \}$ Probe: $7 \cdot (-1) + 3 \cdot 0 - 5 \cdot 1 = -12 \quad (w)$ $-1 \cdot (-1) - 2 \cdot 0 + 4 \cdot 1 = 5 \quad (w)$ $-4 \cdot (-1) + 1 \cdot 0 - 3 \cdot 1 = 1 \quad (w)$
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Hinweis für blutige Anfänger:

Die Vorgehensweise kann in einzelne kleine Schritte zerlegt werden.	
1.	Brüche vermeiden durch zeilenweise Multiplikation mit dem Hauptnenner.
2.	Die erste Zahl in der ersten Zeile soll positiv sein (ev. mit -1 multiplizieren).
3.	Sorgen Sie durch Multiplikation oder Division dafür, dass in der ersten Spalte alle Zahlen den gleichen Betrag haben. In Zeile 2 und 3 soll die erste Zahl negativ sein.
4.	Addiere zur 2. und zur 3. Zeile jeweils die erste. Dadurch entstehen in der ersten Spalte 2 Nullen.
5.	Die zweite Zahl in der 2. Zeile soll positiv sein (ev. mit -1 multiplizieren).
6.	Sorgen Sie durch Multiplikation oder Division dafür, dass ab der 2. Zeile in der zweiten Spalte alle Zahlen den gleichen Betrag haben. In Zeile 3 soll die zweite Zahl negativ sein.
7.	Addieren Sie zur 3. Zeile die 2. Zeile. Dadurch entsteht in der 3. Zeile die 2. Null.
8.	Ermittlung der Lösung durch Rückwärts einsetzen.

Die gleiche Vorgehensweise kann auch auf Systeme mit mehr als drei Gleichungen übertragen werden.

Die Umformungen kann man auch anders durchführen.

Das „wie“ ist ganz dem Geschick des Mathematikers überlassen.

Erst durch intensive Übung gelangt man zu einem optimalen Weg.

Brüche sind möglichst zu vermeiden um keine unnötigen Fehler zu riskieren.

Wer fit ist, kann auch mehrere Umformungen gleichzeitig machen, dadurch ist weniger zu schreiben, die Fehlerquote steigt aber.

tausche: \Leftrightarrow	Ermittlung der Lösung durch Rückwärtseinsetzen
$\begin{array}{ccc c} x_1 & x_2 & x_3 & \\ \hline 7 & 3 & -5 & -12 \\ -1 & -2 & 4 & 5 \\ -4 & 1 & -3 & 1 \\ \hline 1 & 2 & -4 & -5 \\ 7 & 3 & -5 & -12 \\ -4 & 1 & -3 & 1 \\ \hline 1 & 2 & -4 & -5 \\ 0 & -11 & 23 & 23 \\ 0 & 9 & -19 & -19 \\ \hline 1 & 2 & -4 & -5 \\ 0 & -99 & 207 & 207 \\ 0 & 99 & -209 & -209 \\ \hline 1 & 2 & -4 & -5 \\ 0 & -11 & 23 & 23 \\ 0 & 0 & -2 & -2 \end{array}$	$\begin{array}{l} -2x_3 = -2 \\ \Leftrightarrow \underline{x_3 = 1} \\ -11x_2 + 23 \cdot 1 = 23 \\ \Leftrightarrow -11y = 0 \\ \Leftrightarrow \underline{x_2 = 0} \\ x_1 + 2 \cdot 0 - 4 \cdot 1 = -5 \\ \Leftrightarrow \underline{x_1 = -7} \\ \Leftrightarrow \underline{x_1 = -1} \\ L = \{-1; 0; 1\} \end{array}$
	<u>Probe:</u>
	$7 \cdot (-1) + 3 \cdot 0 - 5 \cdot 1 = -12 \quad (w)$
	$-1 \cdot (-1) - 2 \cdot 0 + 4 \cdot 1 = 5 \quad (w)$
	$-4 \cdot (-1) + 1 \cdot 0 - 3 \cdot 1 = 1 \quad (w)$

Beispiel 1: (leicht)

$$2x_1 - 3x_2 + 4x_3 = 8$$

$$3x_1 + 4x_2 - 5x_3 = -4$$

$$4x_1 - 6x_2 + 3x_3 = 1$$

x_1	x_2	x_3		
2	-3	4	8	·6
3	4	-5	-4	·4
4	-6	3	1	·3
12	-18	24	48	:6
12	16	-20	-16	II - I
12	-18	9	3	III - I
2	-3	4	8	
0	34	-44	-64	
0	0	-15	-45	

Ermittlung der Lösung durch
Rückwärtseinsetzen

$$-15x_3 = -45$$

$$\Leftrightarrow \underline{x_3 = 3}$$

$$34x_2 - 44 \cdot 3 = -64$$

$$\Leftrightarrow 34x_2 = 68$$

$$\Leftrightarrow \underline{x_2 = 2}$$

$$2x_1 - 3 \cdot 2 + 4 \cdot 3 = 8$$

$$\Leftrightarrow 2x_1 = 2$$

$$\Leftrightarrow \underline{x_1 = 1}$$

$$L = \{1; 2; 3\}$$

Probe:

$$2 \cdot 1 - 3 \cdot 2 + 4 \cdot 3 = 8 \quad (w)$$

$$3 \cdot 1 + 4 \cdot 2 - 5 \cdot 3 = -4 \quad (w)$$

$$4 \cdot 1 - 6 \cdot 2 + 3 \cdot 3 = 1 \quad (w)$$

Beispiel 2: (mittelschwer)

$3x_1 + 2x_2 - 4x_3 = -2$ $4x_1 - 5x_2 + 3x_3 = 9$ $8x_1 + 7x_2 - 9x_3 = 13$	Ermittlung der Lösung durch Rückwärtseinsetzen																																																																																															
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5; 6 \}$ <p><u>Probe:</u></p> $3 \cdot 4 + 2 \cdot 5 - 4 \cdot 6 = -2 \quad (w)$ $4 \cdot 4 - 5 \cdot 5 + 3 \cdot 6 = 9 \quad (w)$ $8 \cdot 4 + 7 \cdot 5 - 9 \cdot 6 = 13 \quad (w)$
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Beispiel 3: (schwer)

$\frac{1}{2}x_1 - \frac{4}{5}x_2 + \frac{3}{8}x_3 = 4$	x_1	x_2	x_3		
$\frac{3}{4}x_1 + \frac{3}{8}x_2 + \frac{1}{5}x_3 = 23$	$\frac{1}{2}$	$-\frac{4}{5}$	$\frac{3}{8}$	4	·40
$\frac{4}{5}x_1 - \frac{1}{2}x_2 + \frac{1}{4}x_3 = 8$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{5}$	23	·40
Ermittlung der Lösung durch Rückwärtseinsetzen	$\frac{4}{5}$	$-\frac{1}{2}$	$\frac{1}{4}$	8	·20
$-358x_3 = -14320$	20	-32	15	160	·12
$\Leftrightarrow x_3 = 40$	30	15	8	920	·8
$126x_2 - 29 \cdot 40 = 1360$	16	-10	5	160	·15
$\Leftrightarrow 126x_2 = 2520$	240	-384	180	1920	:12
$\Leftrightarrow x_2 = 20$	240	120	64	7360	II - I
$20x_1 - 32 \cdot 20 + 15 \cdot 40 = 160$	240	-150	75	2400	III - I
$\Leftrightarrow 20x_1 = 200$	20	-32	15	160	
$\Leftrightarrow x_1 = 10$	0	504	-116	5440	:4
$L = \{ 10 ; 20 ; 40 \}$	0	234	-105	480	:3
Probe:	20	-32	15	160	
$\frac{1}{2} \cdot 10 - \frac{4}{5} \cdot 20 + \frac{3}{8} \cdot 40 = 4$	0	126	-29	1360	·13
$\Leftrightarrow 5 - 16 + 15 = 4(w)$	0	78	-35	160	·21
$\frac{3}{4} \cdot 10 + \frac{3}{8} \cdot 20 + \frac{1}{5} \cdot 40 = 23$	20	-32	15	160	
$\Leftrightarrow \frac{15}{2} + \frac{15}{2} + 8 = 23(w)$	0	1638	-377	17680	:13
$\frac{4}{5} \cdot 10 - \frac{1}{2} \cdot 20 + \frac{1}{4} \cdot 40 = 8$	0	1638	-735	3360	III - II
$\Leftrightarrow 8 - 10 + 10 = 8(w)$	20	-32	15	160	
	0	126	-29	1360	
	0	0	-358	-14320	

Anwendung des Gauß- Algorithmus zur Berechnung der Funktionsgleichung einer ganzrationalen Funktion von der 4 Punkte bekannt sind.

G1	Ausführlicher Gauß- Algorithmus																																																																																																							
	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$																																																																																																							
	$P_1(1 4) \Rightarrow$	$f(1) = 1a_3 + 1a_2 + 1a_1 + 1a_0 = 4$																																																																																																						
	$P_2(2 2) \Rightarrow$	$f(2) = 8a_3 + 4a_2 + 2a_1 + 1a_0 = 2$																																																																																																						
	$P_3(4 4) \Rightarrow$	$f(4) = 64a_3 + 16a_2 + 4a_1 + 1a_0 = 4$																																																																																																						
	$P_4(5 20) \Rightarrow$	$f(5) = 125a_3 + 25a_2 + 5a_1 + 1a_0 = 20$																																																																																																						
	<table border="1" style="display: inline-table; vertical-align: top;"> <thead> <tr> <th>a_0</th> <th>a_1</th> <th>a_2</th> <th>a_3</th> <th></th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td>1</td> <td>2</td> <td>4</td> <td>8</td> <td>2</td> <td>II - I</td> </tr> <tr> <td>1</td> <td>4</td> <td>16</td> <td>64</td> <td>4</td> <td>III - I</td> </tr> <tr> <td>1</td> <td>5</td> <td>25</td> <td>125</td> <td>20</td> <td>IV - I</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td>0</td> <td>1</td> <td>3</td> <td>7</td> <td>-2</td> <td></td> </tr> <tr> <td>0</td> <td>3</td> <td>15</td> <td>63</td> <td>0</td> <td>III - 3 · II</td> </tr> <tr> <td>0</td> <td>4</td> <td>24</td> <td>124</td> <td>16</td> <td>IV - 4 · II</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td>0</td> <td>1</td> <td>3</td> <td>7</td> <td>-2</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>6</td> <td>42</td> <td>6</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>12</td> <td>96</td> <td>24</td> <td>IV - 2 · III</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td>0</td> <td>1</td> <td>3</td> <td>7</td> <td>-2</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>6</td> <td>42</td> <td>6</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>12</td> <td>12</td> <td></td> </tr> </tbody> </table>	a_0	a_1	a_2	a_3			1	1	1	1	4		1	2	4	8	2	II - I	1	4	16	64	4	III - I	1	5	25	125	20	IV - I	1	1	1	1	4		0	1	3	7	-2		0	3	15	63	0	III - 3 · II	0	4	24	124	16	IV - 4 · II	1	1	1	1	4		0	1	3	7	-2		0	0	6	42	6		0	0	12	96	24	IV - 2 · III	1	1	1	1	4		0	1	3	7	-2		0	0	6	42	6		0	0	0	12	12		$12a_3 = 12 \Leftrightarrow a_3 = 1$ $6a_2 + 42a_3 = 6$ $\Leftrightarrow 6a_2 + 42 = 6 \mid -42$ $\Leftrightarrow 6a_2 = -36 \mid :6 \Leftrightarrow a_2 = -6$ $a_1 + 3a_2 + 7a_3 = -2$ $\Leftrightarrow a_1 - 18 + 7 = -2$ $\Leftrightarrow a_1 - 11 = -2 \mid +11 \Leftrightarrow a_1 = 9$ $a_0 + a_1 + a_2 + a_3 = 4$ $\Leftrightarrow a_0 + 9 - 6 + 1 = 4$ $\Leftrightarrow a_0 + 4 = 4 \mid -4 \Leftrightarrow a_0 = 0$ $f(x) = x^3 - 6x^2 + 9x$
a_0	a_1	a_2	a_3																																																																																																					
1	1	1	1	4																																																																																																				
1	2	4	8	2	II - I																																																																																																			
1	4	16	64	4	III - I																																																																																																			
1	5	25	125	20	IV - I																																																																																																			
1	1	1	1	4																																																																																																				
0	1	3	7	-2																																																																																																				
0	3	15	63	0	III - 3 · II																																																																																																			
0	4	24	124	16	IV - 4 · II																																																																																																			
1	1	1	1	4																																																																																																				
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0	0	6	42	6																																																																																																				
0	0	12	96	24	IV - 2 · III																																																																																																			
1	1	1	1	4																																																																																																				
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0	0	6	42	6																																																																																																				
0	0	0	12	12																																																																																																				

G2		Ausführlicher Gauß- Algorithmus	
$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$			
P_1	$\left(1 \mid -\frac{11}{2}\right)$	$\Rightarrow f(1) = 1a_3 + 1a_2 + 1a_1 + 1a_0 = -\frac{11}{2}$	
P_2	$\left(-1 \mid \frac{9}{2}\right)$	$\Rightarrow f(-1) = -1a_3 + 1a_2 - 1a_1 + 1a_0 = \frac{9}{2}$	
P_3	$\left(-2 \mid 8\right)$	$\Rightarrow f(-2) = -8a_3 + 4a_2 - 2a_1 + 1a_0 = 8$	
P_4	$\left(-3 \mid \frac{5}{2}\right)$	$\Rightarrow f(-3) = -27a_3 + 9a_2 - 3a_1 + 1a_0 = \frac{5}{2}$	
a_0	a_1	a_2	a_3
1	1	1	1
1	-1	1	-1
1	-2	4	-8
1	-3	9	-27
2	2	2	2
2	-2	2	-2
2	-4	8	-16
2	-6	18	-54
2	2	2	2
0	-4	0	-4
0	-6	6	-18
0	-8	16	-56
2	2	2	2
0	2	0	2
0	-2	2	-6
0	-2	4	-14
2	2	2	2
0	2	0	2
0	0	2	-4
0	0	4	-12
2	2	2	2
0	2	0	2
0	0	2	-4
0	0	0	-4

$-\frac{11}{2}$	$ \cdot 2$	
$\frac{9}{2}$	$ \cdot 2$	
8	$ \cdot 2$	$-4a_3 = -4 \Leftrightarrow a_3 = 1$
$\frac{5}{2}$	$ \cdot 2$	
-11		$2a_2 - 4a_3 = -1$
9	II - I	$\Leftrightarrow 2a_2 - 4 = -1 \mid +4$
16	III - I	$\Leftrightarrow 2a_2 = 3 \mid : 2 \Leftrightarrow a_2 = \frac{3}{2}$
5	IV - I	
-11		$2a_1 + 2a_3 = -10$
20	$ \cdot (-2)$	$\Leftrightarrow 2a_1 + 2 = -10 \mid -2$
27	$ \cdot 3$	$\Leftrightarrow 2a_1 = -12 \mid : 2 \Leftrightarrow a_1 = -6$
16	$ \cdot 4$	
-11		$2a_0 + 2a_1 + 2a_2 + 2a_3 = -11$
-10		$\Leftrightarrow 2a_0 - 12 + 3 + 2 = -11$
9	III + II	$\Leftrightarrow 2a_0 - 7 = -11 \mid +7$
4	IV + II	$\Leftrightarrow 2a_0 = -4 \mid : 2 \Leftrightarrow a_0 = -2$
-11		
-10		
-1		
-6	IV - 2 \cdot III	$f(x) = x^3 + \frac{3}{2}x^2 - 6x - 2$
-11		
-10		
-1		
-4		

G4	Ausführlicher Gauß- Algorithmus				
$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$					
$P_1(-1 7) \Rightarrow f(-1) = -1a_3 + 1a_2 - 1a_1 + 1a_0 = 7$					
$P_2(-2 6) \Rightarrow f(-2) = -8a_3 + 4a_2 - 2a_1 + 1a_0 = 6$					
$P_3(3 1) \Rightarrow f(3) = 27a_3 + 9a_2 + 3a_1 + 1a_0 = 1$					
$P_4(-3 -2) \Rightarrow f(-3) = -27a_3 + 9a_2 - 3a_1 + 1a_0 = -2$					
$20a_2 = -10 \Leftrightarrow a_2 = -\frac{1}{2}$					
a_0	a_1	a_2	a_3		
1	-1	1	-1	7	
1	-2	4	-8	6	II - I
1	3	9	27	1	III - I
1	-3	9	-27	-2	IV - I
1	-1	1	-1	7	
0	-1	3	-7	-1	$\Rightarrow \Leftrightarrow -a_1 - \frac{3}{2} - \frac{7}{2} = -1$
0	4	8	28	-6	III + 4 · II
0	-2	8	-26	-9	IV - 2 · II
1	-1	1	-1	7	$\Leftrightarrow -a_1 = 4 \quad :(-1) \Leftrightarrow a_1 = -4$
0	-1	3	-7	-1	$a_0 - a_1 + a_2 - a_3 = 7$
0	0	20	0	-10	$\Leftrightarrow a_0 + 4 - \frac{1}{2} - \frac{1}{2} = 7$
0	0	2	-12	-7	$\Leftrightarrow a_0 + 3 = 7 \quad -3 \Leftrightarrow a_0 = 4$
Die Funktionsgleichung: $f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 - 4x + 4$					

G5		Ausführlicher Gauß- Algorithmus	
$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$			
$P_1(2 22) \Rightarrow$	$f(2) =$	$8a_3 + 4a_2 + 2a_1 + 1a_0 =$	22
$P_2(4 44) \Rightarrow$	$f(4) =$	$64a_3 + 16a_2 + 4a_1 + 1a_0 =$	44
$P_3(-4 4) \Rightarrow$	$f(-4) =$	$-64a_3 + 16a_2 - 4a_1 + 1a_0 =$	4
$P_4(8 40) \Rightarrow$	$f(8) =$	$512a_3 + 64a_2 + 8a_1 + 1a_0 =$	40
a_0	a_1	a_2	a_3
1	2	4	8
1	4	16	64
1	-4	16	-64
1	8	64	512
22			
44	II - I		
4	III - I		
40	IV - I		
22			
22			
-18	III + 3 · II		
18	IV - 3 · II		
22			
22			
48			
-48	IV - $\frac{1}{2}$ · III		
22			
22			
48			
-72			
22			
22			
48			
-72			
Die Funktionsgleichung:			
$f(x) = -\frac{1}{4}x^3 + \frac{3}{2}x^2 + 9x$			

G7		Ausführlicher Gauß- Algorithmus	
$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$			
$P_1(1 1) \Rightarrow$	$f(1) =$	$1a_3 + 1a_2 + 1a_1 + 1a_0 =$	1
$P_2(2 0) \Rightarrow$	$f(2) =$	$8a_3 + 4a_2 + 2a_1 + 1a_0 =$	0
$P_3(-2 4) \Rightarrow$	$f(-2) =$	$-8a_3 + 4a_2 - 2a_1 + 1a_0 =$	4
$P_4(3 9) \Rightarrow$	$f(3) =$	$27a_3 + 9a_2 + 3a_1 + 1a_0 =$	9
a_0	a_1	a_2	a_3
1	1	1	1
1	2	4	8
1	-2	4	-8
1	3	9	27
1	1	1	1
0	1	3	7
0	-3	3	-9
0	2	8	26
1	1	1	1
0	1	3	-1
0	0	12	12
0	0	2	12
1	1	1	1
0	1	3	-1
0	0	12	12
0	0	0	10

						$10a_3 = 10 \Leftrightarrow a_3 = 1$
					II - I	
					III - I	$12a_2 + 12a_3 = 0$
					IV - I	$\Leftrightarrow 12a_2 + 12 = 0 \mid -12$
						$\Leftrightarrow 12a_2 = -12 \mid :12 \Leftrightarrow a_2 = -1$
					III + 3 · II	$a_1 + 3a_2 + 7a_3 = -1$
					IV - 2 · II	$\Leftrightarrow a_1 - 3 + 7 = -1$
						$\Leftrightarrow a_1 + 4 = -1 \mid -4 \Leftrightarrow a_1 = -5$
					IV - $\frac{1}{6}$ · III	$a_0 + a_1 + a_2 + a_3 = 1$
						$\Leftrightarrow a_0 - 5 - 1 + 1 = 1$
						$\Leftrightarrow a_0 - 5 = 1 \mid +5 \Leftrightarrow a_0 = 6$
						$f(x) = x^3 - x^2 - 5x + 6$

G9		Ausführlicher Gauß-Algorithmus	
$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$			
P_1	$\left(1 \mid -\frac{9}{2}\right)$	$\Rightarrow f(1) = 1a_3 + 1a_2 + 1a_1 + 1a_0 = -\frac{9}{2}$	
P_2	$\left(-1 \mid \frac{11}{2}\right)$	$\Rightarrow f(-1) = -1a_3 + 1a_2 - 1a_1 + 1a_0 = \frac{11}{2}$	
P_3	$\left(3 \mid -\frac{5}{2}\right)$	$\Rightarrow f(-3) = 27a_3 + 9a_2 + 3a_1 + 1a_0 = -\frac{5}{2}$	
P_4	$\left(-\frac{5}{2} \mid -8\right)$	$\Rightarrow f\left(-\frac{5}{2}\right) = -\frac{125}{8}a_3 + \frac{25}{4}a_2 - \frac{5}{2}a_1 + 1a_0 = -8$	
a_0	a_1	a_2	a_3
1	1	1	1
1	-1	1	-1
1	3	9	27
1	$-\frac{5}{2}$	$\frac{25}{4}$	$-\frac{125}{8}$
8	8	8	8
8	-8	8	-8
8	24	72	216
8	-20	50	-125
8	8	8	8
0	-16	0	-16
0	16	64	208
0	-28	42	-133
2	2	2	2
0	-4	0	-4
0	4	16	52
0	-4	6	-19
2	2	2	2
0	-4	0	-4
0	0	16	48
0	0	6	-15
2	2	2	2
0	-4	0	-4
0	0	2	6
0	0	2	-5
2	2	2	2
0	-4	0	-4
0	0	2	6
0	0	0	-11

$-\frac{9}{2} \quad \cdot 8$ $\frac{11}{2} \quad \cdot 8$ $-\frac{5}{2} \quad \cdot 8$ $-8 \quad \cdot 8$ -36 $44 \quad \text{II} - \text{I}$ $-20 \quad \text{III} - \text{I}$ $-64 \quad \text{IV} - \text{I}$ $-36 \quad :4$ $80 \quad :4$ $16 \quad :4$ $-28 \quad :7$ -9 20 $4 \quad \text{III} + \text{II}$ $-4 \quad \text{IV} - \text{II}$ -9 20 $24 \quad :8$ $-24 \quad :3$ -9 20 3 $-8 \quad \text{IV} - \text{III}$ -9 20 3 -11	$-11a_3 = -11 \quad :(-11)$ $\Leftrightarrow a_3 = 1$ $2a_2 + 6a_3 = 3$ $\Leftrightarrow 2a_2 + 6 = 3 \quad -6$ $\Leftrightarrow 2a_2 = -3 \quad :2$ $\Leftrightarrow a_2 = -\frac{3}{2}$ $-4a_1 - 4a_3 = 20$ $\Leftrightarrow -4a_1 - 4 = 20 \quad +4$ $\Leftrightarrow -4a_1 = 24 \quad :(-4)$ $\Leftrightarrow a_1 = -6$ $2a_0 + 2a_1 + 2a_2 + 2a_3 = -9$ $\Leftrightarrow 2a_0 - 12 - 3 + 2 = -9$ $\Leftrightarrow 2a_0 - 13 = -9 \quad +13$ $\Leftrightarrow 2a_0 = 4 \quad :2$ $\Leftrightarrow a_0 = 2$ $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2$
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G10		Ausführlicher Gauß- Algorithmus	
$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$			
$P_1(1 25) \Rightarrow$	$f(1) =$	$1a_3 + 1a_2 + 1a_1 + 1a_0 =$	25
$P_2(-1 -49) \Rightarrow$	$f(-1) =$	$-1a_3 + 1a_2 - 1a_1 + 1a_0 =$	-49
$P_3(3 27) \Rightarrow$	$f(3) =$	$27a_3 + 9a_2 + 3a_1 + 1a_0 =$	27
$P_4(5 5) \Rightarrow$	$f(5) =$	$125a_3 + 25a_2 + 5a_1 + 1a_0 =$	5
a_0	a_1	a_2	a_3
1	1	1	1
1	-1	1	-1
1	3	9	27
1	5	25	125
25			
-49	II-I		
27	III-I		
5	IV-I		
25			
0	-2	0	-2
0	2	8	26
0	4	24	124
-74	III+II		
-20	IV+2·II		
25			
0	-2	0	-2
0	0	8	24
0	0	24	120
-168	IV-3·III		
25			
0	-2	0	-2
0	0	8	24
0	0	0	48
-74			
-72			
48			
$48a_3 = 48 \Leftrightarrow a_3 = 1$			
$8a_2 + 24a_3 = -72$			
$\Leftrightarrow 8a_2 + 24 = -72 \mid -24$			
$\Leftrightarrow 8a_2 = -96 \mid :8 \Leftrightarrow a_2 = -12$			
$-2a_1 - 2a_3 = -74$			
$\Leftrightarrow -2a_1 - 2 = -74 \mid +2$			
$\Leftrightarrow -2a_1 = -72 \mid :(-2) \Leftrightarrow a_1 = 36$			
$a_0 + a_1 + a_2 + a_3 = 25$			
$\Leftrightarrow a_0 + 36 - 12 + 1 = 25$			
$\Leftrightarrow a_0 + 25 = 25 \mid -25 \Leftrightarrow a_0 = 0$			
$f(x) = x^3 - 12x^2 + 36x$			

Aufgaben zur Übung

Zur Ergebniskontrolle ist die Probe durchzuführen

1.	$\begin{cases} x_1 + x_2 + 0x_3 = 28 \\ x_1 + 0x_2 + x_3 = 30 \\ 0x_1 + x_2 + x_3 = 32 \end{cases} \Rightarrow L = \{13; 15; 17\}$	2.	$\begin{cases} 2x_1 + 3x_2 + 0x_3 = 12 \\ 3x_1 + 0x_2 + 2x_3 = 11 \\ 0x_1 + 3x_2 + 4x_3 = 10 \end{cases} \Rightarrow L = \{3; 2; 1\}$																									
3.	$\begin{cases} x_1 + x_2 - x_3 = 17 \\ x_1 - x_2 + x_3 = 13 \\ -x_1 + x_2 + x_3 = 7 \end{cases} \Rightarrow L = \{15; 12; 10\}$	4.	$\begin{cases} -4x_1 + 3x_2 - 2x_3 = 8 \\ 5x_1 + 4x_2 - 6x_3 = -8 \\ -3x_1 + 2x_2 + 4x_3 = 19 \end{cases} \Rightarrow L = \{-1; 3; 2,5\}$																									
5.	$\begin{cases} x_1 + x_2 + x_3 = 9 \\ x_1 + 2x_2 + 4x_3 = 15 \\ x_1 + 3x_2 + 9x_3 = 23 \end{cases} \Rightarrow L = \{5; 3; 1\}$	6.	$\begin{cases} x_1 + x_2 + x_3 = 9 \\ x_1 + 2x_2 + 3x_3 = 14 \\ x_1 + 3x_2 + 6x_3 = 20 \end{cases} \Rightarrow L = \{5; 3; 1\}$																									
7.	$\begin{cases} x_1 + 3x_2 - 2x_3 + x_4 = -7 \\ -2x_1 + x_2 - 4x_3 - 5x_4 = -6 \\ x_1 - 3x_2 + x_3 = 6 \\ -3x_1 + 4x_2 - 6x_3 + 2x_4 = -21 \end{cases} \Rightarrow L = \{1; -1; 2; -1\}$	<table border="1"> <thead> <tr> <th>x_1</th> <th>x_2</th> <th>x_3</th> <th>x_4</th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> <td>-2</td> <td>1</td> <td>-7</td> </tr> <tr> <td>-2</td> <td>1</td> <td>-4</td> <td>-5</td> <td>-6</td> </tr> <tr> <td>1</td> <td>-3</td> <td>1</td> <td>0</td> <td>6</td> </tr> <tr> <td>-3</td> <td>4</td> <td>-6</td> <td>2</td> <td>-21</td> </tr> </tbody> </table>		x_1	x_2	x_3	x_4		1	3	-2	1	-7	-2	1	-4	-5	-6	1	-3	1	0	6	-3	4	-6	2	-21
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8.	$\begin{cases} -2x_1 + 3x_2 - 4x_3 + 2x_4 = 9 \\ 3x_1 + 4x_2 - 8x_3 + x_4 = -2 \\ x_1 - 6x_2 - x_3 + 3x_4 = 20 \\ x_4 = 5 \end{cases} \Rightarrow L = \{-1; -1; 0; 5\}$	<table border="1"> <thead> <tr> <th>x_1</th> <th>x_2</th> <th>x_3</th> <th>x_4</th> <th></th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>3</td> <td>-4</td> <td>2</td> <td>9</td> </tr> <tr> <td>3</td> <td>4</td> <td>-8</td> <td>1</td> <td>-2</td> </tr> <tr> <td>1</td> <td>-6</td> <td>-1</td> <td>3</td> <td>20</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>5</td> </tr> </tbody> </table>		x_1	x_2	x_3	x_4		-2	3	-4	2	9	3	4	-8	1	-2	1	-6	-1	3	20	0	0	0	1	5
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