Lösungen Training Integralrechnung II

Ergebnisse:

E1	Ergebnis
	$\int_{1}^{3} x\,dx = \left[\frac{1}{2}x^2\right]_1^3 = \underline{\underline{4}}$

E2	Ergebnis
	$\int_{0}^{3} \left(x^2 - 1\right)dx = \left[\frac{1}{3}x^3 - x\right]_0^3 = \underline{\underline{6}}$

E3	Ergebnis
	$\int_{-2}^{2} 4\,dx = \left[4x\right]_{-2}^{2} = \underline{\underline{16}}$

E4	Ergebnis
	$\int_{3}^{4} dx = \left[x\right]_3^4 = \underline{\underline{1}}$

E5	Ergebnis
	$\int_{0}^{4} (2x - 5)\,dx = \left[x^2 - 5x\right]_0^4 = \underline{\underline{-4}}$

E6	Ergebnis
	$\int_{-\sqrt{8}}^{\sqrt{8}} \left(\frac{1}{2}x^2 - 4\right)dx = \left[\frac{1}{6}x^3 - 4x\right]_{-\sqrt{8}}^{\sqrt{8}} \approx \underline{\underline{-15{,}085}}$

E7	Ergebnis
	$\int_{-3}^{3} \left(x^3 + 2x\right)dx = \left[\frac{1}{4}x^4 + x^2\right]_{-3}^{3} = \underline{\underline{0}}$

E8	Ergebnis
	$\int_{-1}^{2} \left(x^3 - \frac{1}{2}x^2 + 3x - 4\right)dx = \left[\frac{1}{4}x^4 - \frac{1}{6}x^3 + \frac{3}{2}x^2 - 4x\right]_{-1}^{2} = \underline{\underline{-\frac{21}{4}}}$

E9	Ergebnis
	$\int_{-4}^{4} \left(2x^2 - \frac{1}{8}x^4\right)dx = \left[\frac{2}{3}x^3 - \frac{1}{40}x^5\right]_{-4}^{4} = \underline{\underline{\frac{512}{15}}}$

E10	Ergebnis
	$\int_2^3 (3x-6)^3\,dx = \left[\frac{27}{4}x^4 - \frac{162}{3}x^3 + \frac{324}{2}x^2 - 216x\right]_2^3 = \underline{\underline{\frac{27}{4}}}$

Ausführliche Lösung:

A1	Ausführliche Lösung
	$\int_1^3 x\,dx = \left[\frac{1}{2}x^2\right]_1^3 = \frac{1}{2}\cdot 3^2 - \frac{1}{2}\cdot 1^2 = \frac{1}{2}\cdot 9 - \frac{1}{2}\cdot 1 = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = \underline{\underline{4}}$

A2	Ausführliche Lösung
	$\int_0^3 \left(x^2 - 1\right) dx = \left[\frac{1}{3}x^3 - x\right]_0^3 = \frac{1}{3}\cdot 3^3 - 3 - \left(\frac{1}{3}\cdot 0^3 - 0\right) = 3^2 - 3 - (0) = 9 - 3 = \underline{\underline{6}}$

A3	Ausführliche Lösung
	$\int_{-2}^2 4\,dx = [4x]_{-2}^2 = 4\cdot 2 - 4\cdot(-2) = 8 + 8 = \underline{\underline{16}}$

A4	Ausführliche Lösung
	$\int_3^4 dx = [x]_3^4 = 4 - 3 = \underline{\underline{1}}$

A5	Ausführliche Lösung
	$\int_0^4 (2x - 5)\,dx = \left[x^2 - 5x\right]_0^4 = 4^2 - 5\cdot 4 - \left(0^2 - 5\cdot 0\right) = 16 - 20 - 0 = \underline{\underline{-4}}$

A6	Ausführliche Lösung
	$\int_{-\sqrt{8}}^{\sqrt{8}} \left(\frac{1}{2}x^2 - 4\right) dx = \left[\frac{1}{6}x^3 - 4x\right]_{-\sqrt{8}}^{\sqrt{8}} = \frac{1}{6}\cdot\left(\sqrt{8}\right)^3 - 4\cdot\sqrt{8} - \left(\frac{1}{6}\cdot\left(-\sqrt{8}\right)^3 - 4\cdot\left(-\sqrt{8}\right)\right)$
	$= \frac{1}{6}\cdot 8\cdot\sqrt{8} - 4\cdot\sqrt{8} - \left(-\frac{1}{6}\cdot 8\cdot\sqrt{8} + 4\cdot\sqrt{8}\right) = \frac{8}{6}\cdot\sqrt{8} - \frac{24}{6}\cdot\sqrt{8} + \frac{8}{6}\cdot\sqrt{8} - \frac{24}{6}\cdot\sqrt{8}$
	$= -\frac{32}{6}\cdot\sqrt{8} = \underline{\underline{-\frac{16}{3}\cdot\sqrt{8} \approx -15{,}085}}$

A7	Ausführliche Lösung
	$\int_{-3}^3 \left(x^3 + 2x\right) dx = \left[\frac{1}{4}x^4 + x^2\right]_{-3}^3 = \frac{1}{4}\cdot 3^4 + 3^2 - \left(\frac{1}{4}\cdot(-3)^4 + (-3)^2\right)$
	$= \frac{81}{4} + 9 - \left(\frac{81}{4} + 9\right) = \underline{\underline{0}}$

A8 Ausführliche Lösung

$$\int_{-1}^{2}\left(x^3-\frac{1}{2}x^2+3x-4\right)dx=\left[\frac{1}{4}x^4-\frac{1}{6}x^3+\frac{3}{2}x^2-4x\right]_{-1}^{2}$$

$$=\frac{1}{4}\cdot 2^4-\frac{1}{6}\cdot 2^3+\frac{3}{2}\cdot 2^2-4\cdot 2-\left[\frac{1}{4}\cdot(-1)^4-\frac{1}{6}\cdot(-1)^3+\frac{3}{2}\cdot(-1)^2-4\cdot(-1)\right]$$

$$=\frac{16}{4}-\frac{8}{6}+\frac{12}{2}-8-\left[\frac{1}{4}\cdot 1-\frac{1}{6}\cdot(-1)+\frac{3}{2}\cdot 1+4\right]$$

$$=4-\frac{4}{3}+6-8-\left[\frac{1}{4}+\frac{1}{6}+\frac{3}{2}+4\right]$$

$$=2-\frac{4}{3}-\frac{1}{4}-\frac{1}{6}-\frac{3}{2}-4=-\frac{24}{12}-\frac{16}{12}-\frac{3}{12}-\frac{2}{12}-\frac{18}{12}=-\frac{63}{12}=-\frac{21}{4}$$

A9 Ausführliche Lösung

$$\int_{-4}^{4}\left(2x^2-\frac{1}{8}x^4\right)dx=\left[\frac{2}{3}x^3-\frac{1}{40}x^5\right]_{-4}^{4}=\frac{2}{3}\cdot 4^3-\frac{1}{40}\cdot 4^5-\left[\frac{2}{3}\cdot(-4)^3-\frac{1}{40}\cdot(-4)^5\right]$$

$$=\frac{128}{3}-\frac{1024}{40}-\left[\frac{2}{3}\cdot(-64)-\frac{1}{40}\cdot(-1024)\right]$$

$$=\frac{128}{3}-\frac{1024}{40}-\left[-\frac{128}{3}+\frac{1024}{40}\right]=\frac{128}{3}-\frac{1024}{40}+\frac{128}{3}-\frac{1024}{40}$$

$$=\frac{256}{3}-\frac{2048}{40}=\frac{256}{3}-\frac{256}{5}=\frac{1280}{15}-\frac{768}{15}=\frac{512}{15}$$

A10 Ausführliche Lösung

$$\int_{2}^{3}(3x-6)^3\,dx=\int_{2}^{3}(3x-6)^2\cdot(3x-6)\,dx=\int_{2}^{3}\left(9x^2-36x+36\right)\cdot(3x-6)\,dx$$

$$=\int_{2}^{3}\left(27x^3-162x^2+324x-216\right)dx$$

$$=\left[\frac{27}{4}x^4-\frac{162}{3}x^3+\frac{324}{2}x^2-216x\right]_{2}^{3}$$

$$=\frac{27}{4}\cdot 3^4-\frac{162}{3}\cdot 3^3+\frac{324}{2}\cdot 3^2-216\cdot 3-\left[\frac{27}{4}\cdot 2^4-\frac{162}{3}\cdot 2^3+\frac{324}{2}\cdot 2^2-216\cdot 2\right]$$

$$=\frac{2187}{4}-1458+\frac{2916}{2}-648-108+\frac{1296}{3}-648+432$$

$$=\frac{2187}{4}-1458+1458-648-108+432-648+432$$

$$=\frac{2187}{4}-540=\frac{2187}{4}-\frac{2160}{4}=\frac{27}{4}$$